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PROPAGATION OF PRESSURE WAVES IN A GAS–LIQUID MEDIUM WITH A CLUSTER STRUCTURE

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Propagation of a stepwise shock wave in a liquid containing spherical gas–liquid clusters is experimentally studied. Measured results are compared with available theoretical models. It is shown that resonant interaction of gas–liquid clusters in the wave can increase the amplitude of oscillations in the shock wave.

Key words: *shock wave, gas–liquid cluster.*

Propagation of pressure waves in a liquid with gas bubbles was considered in much detail both theoretically and experimentally [1–9]. It was shown that a nonlinear finite-length disturbance in a liquid with gas bubbles decomposes into solitary waves (solitons). Shock waves in bubbly media can have an oscillating structure. Wave evolution, structure, and decay were studied. A new type of wave structures (multisolitons) was found and examined in the experiments of [10–12] in a liquid with gas bubbles of two different sizes with different ratios of bubble radii. The influence of inhomogeneity of the gas–liquid mixture and compressibility of the liquid on the pressure-wave structure was examined in [13, 14]. The structure and decay of moderate-amplitude pressure waves in a liquid with bubbles of two different gases and in bubbly media with a stratified structure were experimentally studied in [15, 16]. Generation of high-power pressure pulses by spherical bubbly clusters was numerically considered by Kedrinskii et al. [17] who were the first to suggest the problem formulation and to explain the mechanism of shock-wave amplification by a spherical bubbly cluster. Interaction of a two-dimensional shock wave with a spherical bubbly cluster in a liquid was experimentally studied in [18].

In the present work, we experimentally examined the evolution and structure of a moderate-amplitude shock wave in a liquid containing bubbly clusters.

The experiments were performed on a "shock-tube" setup (Fig. 1). The test section was a vertical thick-walled steel tube with an inner diameter of 53 mm and a length of 1 m. A stainless-steel wire 1 mm in diameter was located at the axis along the test section; the ends of the wire were attached to the test-section walls. The test section was partly filled by a liquid under vacuum, which allowed us to avoid formation of gas bubbles in the liquid. Distilled water was used as the test liquid. In the test section, water was saturated by air to the equilibrium state at room temperature and atmospheric pressure; the experiments were performed under these conditions. Five bubbly clusters (foam-rubber spheres 30 mm in diameter filled by the liquid with gas bubbles) were spit onto the wire by their centers. The upper edge of the upper cluster was located at a distance of 10 mm from the liquid surface. The centers of the remaining clusters were located exactly opposite the pressure gauges G2–G5.

The bubbly clusters were prepared on an additional setup as follows. The foam-rubber bubbles were placed into the test volume of the setup and were saturated by distilled water under vacuum. After that air bubbles with an elevated static pressure (as compared to the atmospheric value) were pumped through the liquid in the test section. The liquid in the test volume was saturated by air to the equilibrium state at a given static pressure, and the gas dissolved inside the foam-rubber spheres owing to diffusion. The time needed for equalization of concentrations of the gas dissolved in the liquid on the sphere surface and in its center was $\tau \approx 15$ h ($\tau \approx R^2/(2D)$ [19], R is the

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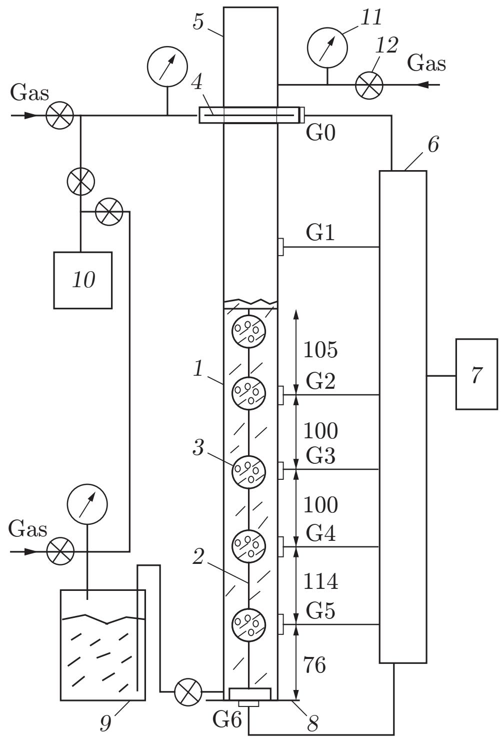

Fig. 1. Layout of the setup: 1) test section; 2) thin wire; 3) gas–liquid cluster; 4) diaphragm; 5) high-pressure chamber; 6) analog-to-digital converter; 7) computer; 8) bottom of the test section; 9) tank for the liquid; 10) vacuum pump; 11) manometers and compound pressure-vacuum gauges; 12) taps; the pressure gauges are indicated by G0–G6.

cluster radius, and D is the diffusion coefficient). When the static pressure was reduced to the atmospheric value, gas bubbles evolved from the liquid. The bubbles stuck to the foam-rubber skeleton and formed the gas–liquid cluster.

Note, the porosity of the foam-rubber sphere is rather high (about 98%), and its rigidity is low; hence, the porous skeleton does not affect pressure-wave propagation [20].

Assuming that the process of incipience of gas bubbles in the cluster after the decrease in static pressure is heterogeneous, which is valid for commonly used distilled water, we can evaluate the critical radius of the nucleus from which the bubble starts to grow and the size of the bubble to which the nucleus grows after the decrease in static pressure [4, 20]. For air bubbles, the diameter is $d \approx 10^{-4}$ m. Yet, gas bubbles up to $d \approx 5 \cdot 10^{-4}$ m were observed on the cluster surface, which could be caused by coalescence of bubbles in the course of their growth after the decrease in static pressure.

By changing the decrease in static pressure, one can change the initial volume content of the gas in clusters φ_c. The mean volume content of the gas in clusters was determined by the increase in volume of the liquid due to the decrease in the initial static pressure in the medium and by the volume of clusters [20].

Stepwise pressure waves originated in air owing to breakdown of the diaphragm separating the high-pressure chamber and the test section and propagated into the liquid. The pressure-wave profiles were registered by piezoelectric pressure gauges located on the side surface (G1–G5) and on the test-section bottom (G6); the gauges were flush-mounted on the inner surface of the test section. The gauge G0 was used to trigger the analog-to-digital converter (ADC). The signals from the gauges were fed to the ADC and then were processed on a computer.

Figure 2 shows the time evolution of the pressure waves at different x distances from the point where the shock wave enters the medium for different initial amplitudes of the waves and initial volume contents of the gas in the clusters. Here, ΔP_0 is the amplitude of the shock wave entering the liquid. It equals the amplitude of the air shock wave reflected from the liquid surface (Fig. 2a; $x < 0$); $\Delta P_{\max}$ is the amplitude of the first oscillation of the pressure wave in the liquid; P_0 is the initial static pressure in the medium. The wave amplitudes are indicated above

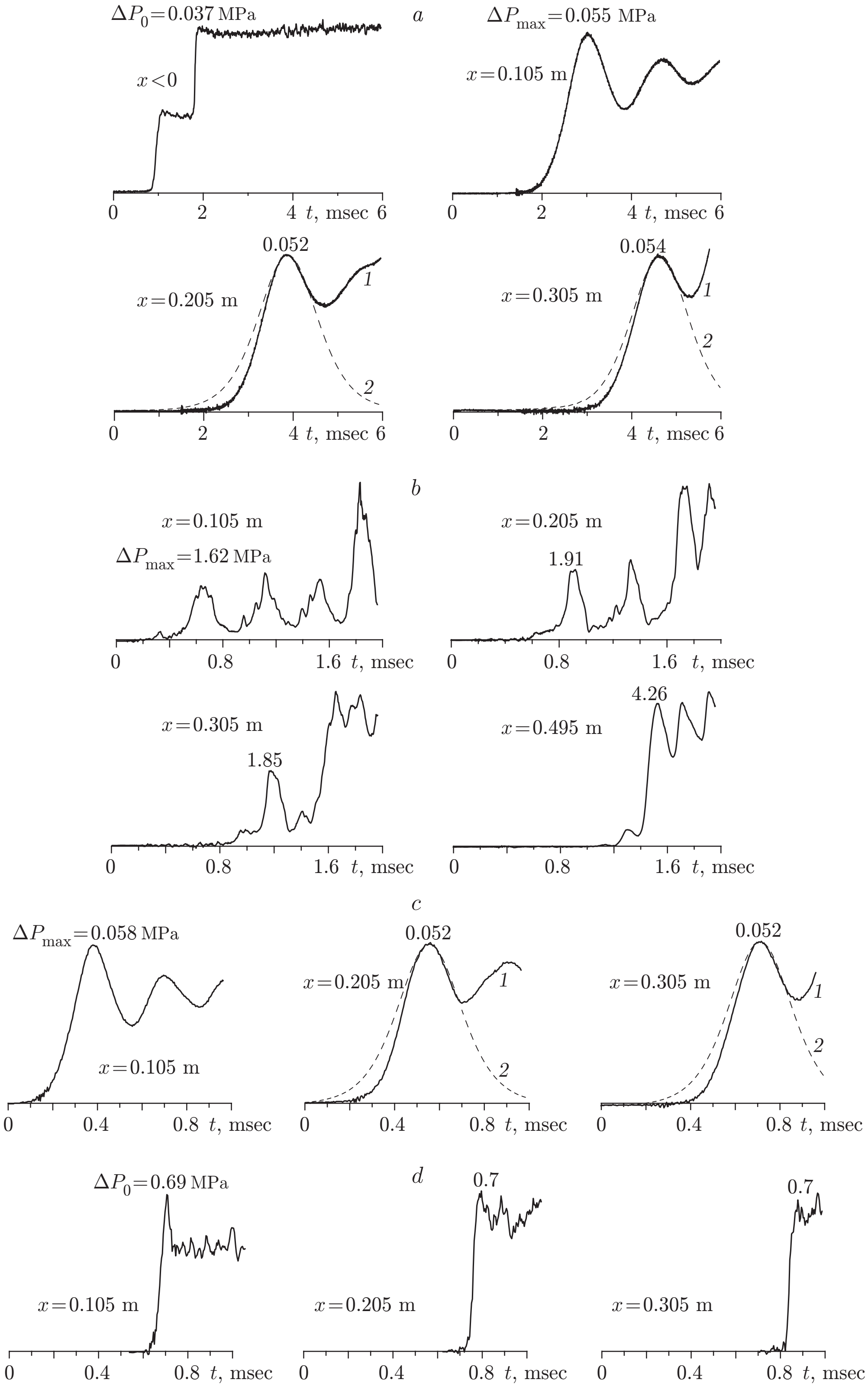

Fig. 2. Evolution of the shock wave in a gas–liquid medium with a cluster structure: (a, b) P_0 = 0.101 MPa, φ_0 = 7.5%, φ_c = 9.8%, and ΔP_0 = 0.037 (a) and 0.83 MPa (b); (c, d) P_0 = 0.1 MPa, φ_0 = 7.5%, φ_c = 0.38%, and ΔP_0 = 0.037 (c) and 0.69 MPa (d); curves 1 refer to the experimental data and curves 2 refer to the calculations by Eqs. (1) and (2).

the wave profiles, and the time scale is shown on the abscissa axis. The gauge at a distance $x = 0.495$ m (Fig. 2b) from the point of the wave entrance into the medium is flush-mounted into the test-section bottom and registers the wave reflected from the bottom. It is seen that the initial stepwise wave is transformed into an oscillating shock wave already at the second bubbly cluster (Fig. 2a; $x = 0.105$ m). As for a homogeneous bubbly mixture [3], the amplitude of the first oscillation is 1.5 times greater than the mean pressure in the wave for low-amplitude waves. Figure 2a ($x = 0.205$ and 0.305 m) shows a comparison of the shape of the first oscillation of the shock wave (curve 1) with the shape of the soliton of an identical amplitude calculated by the Boussinesq equation for a gas–liquid medium (curve 2) [3]:

$$\Delta P(t) = \Delta P_{\max} \sec h^2 (t/\Delta t_{\rm B}); \tag{1}$$

$$\Delta t_{\rm B} = (6\beta/(B\Delta P_{\max}))^{0.5}. \tag{2}$$

Here $\Delta t_{\rm B}$ is the half-width of the calculated Boussinesq soliton, equal to the wave length from the pressure of $0.42\Delta P_{\max}$ to the maximum value of pressure in the wave $\Delta P_{\max}$; β and B are the dispersion and nonlinearity coefficients, respectively. For a gas–liquid medium with a cluster structure, the expressions for the coefficients in the case of an isothermal behavior of the gas in the bubbles have the form [21]

$$\beta = R^2\,\frac{1 + 0.2(1 - \varphi_c)}{3\varphi_0(1 - \varphi_0\varphi_c)}, \qquad B = \frac{1 - \varphi_0\varphi_c}{(\rho_1(1 - \varphi_0) + \rho_2\varphi_0)\varphi_0\varphi_c},$$

where φ_0 is the volume content of clusters in the medium; ρ_1 and ρ_2 are the densities of the liquid and gas, respectively. In contrast to a homogeneous bubbly medium, the dispersion in a gas–liquid medium with a cluster structure is determined by the cluster radius R and the volume content of clusters in the medium φ_0 [21]. As for homogeneous bubbly media [3], the shape of the first oscillation of the shock wave is similar to the shape of the soliton for low wave amplitudes. Deviation of the experimental profile from the predicted soliton shape can be caused by two reasons. First, probably, the oscillating shock wave has not been completely formed at these distances. Second, the condition of the long-wave approximation for the Boussinesq equation in gas–liquid media (namely, the wave length should be much greater than the cluster size and the distance between the clusters) was not satisfied in the experiments.

With increasing amplitude of the wave entering the medium, the duration of oscillations decreases, and the oscillating shock wave starts decomposing into solitary waves. The solitary waves in Fig. 2b are still connected to each other, and their amplitude is almost twice as high as the mean pressure in the incident shock wave. As a result of nonlinear reflection from the rigid bottom, the wave amplitude substantially increases (see Fig. 2b; $x = 0.495$ m). A typical feature of high-amplitude waves is the formation of a precursor ahead of the main signal (Fig. 2b; $x = 0.205$ and 0.305 m). Note that this precursor differs from the precursor in bubbly media (the latter propagates with a velocity of sound in a pure liquid). The precursor in the present work is formed, apparently, by high-frequency oscillations formed owing to nonlinear oscillations of clusters in the liquid and having a higher velocity than the velocity of the main wave. The precursor velocity is close to the adiabatic value, whereas the main wave velocity, as is shown below, equals the isothermal velocity.

As the initial volume content of the gas in the cluster decreases, the duration of oscillations in the oscillating shock wave decreases for identical wave amplitudes (Figs. 2c and 2a), whereas the duration of oscillations in a homogeneous bubbly structure is independent of the volume gas content and is determined only by the bubble size and wave amplitude [3]. Curve 2 in Fig. 2c shows the predicted shape of the soliton. The structure of the first oscillation of the shock wave is close to the soliton shape for low wave amplitudes.

As the wave amplitude increases in a medium with a low initial volume content of the gas in the cluster, the effect of nonlinearity prevails over dispersion, and the wave entering the medium practically retains its stepwise shape during its propagation (Fig. 2d).

Figure 3 shows the shock-wave velocity U in a gas–liquid medium with a cluster structure as a function of shock-wave amplitude (c_0 is the velocity of sound in the gas–liquid medium and c_1 is the velocity of sound in the liquid). Points 1 and 2 refer to the experimental values of shock-wave velocity in a medium with a rather high volume content of the gas, where the compressibility of the liquid does not exert any significant effect on wave velocity. The shock-wave velocity was determined by the time of arrival of the peak of the first oscillation at two neighboring gauges (G3 and G4) and by the distance between the gauges. The velocity of sound in the

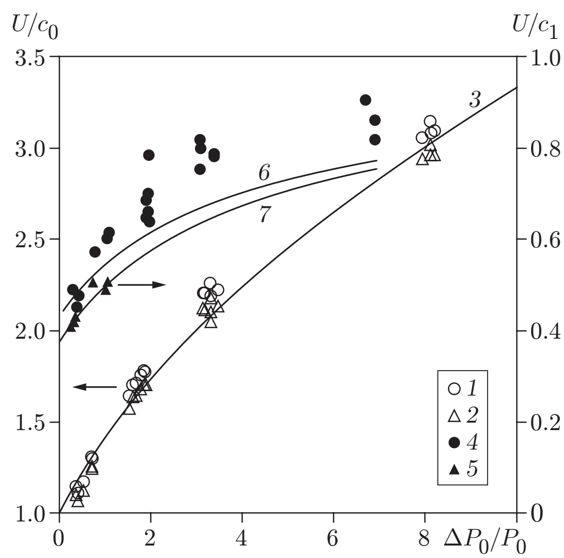

Fig. 3. Shock-wave velocity versus its amplitude for $\varphi_0 = 7.5\%$ and $\varphi_c = 8.4$–9.8% (points 1 and 2 and curve 3) and $\varphi_c = 0.38\%$ (points 4 and 5 and curves 6 and 7): points 1, 2, 4, and 5 refer to the experimental data, curve 3 refer to the calculation by model (5), and curves 6 and 7 refer to the calculations by models (6) and (7), respectively.

gas–liquid medium with a cluster structure for the isothermal behavior of the gas in the bubbles was calculated by the homogeneous [3] and cluster [4] models, respectively:

$$c_0 = \Big(\frac{P_0}{(\rho_1(1-\varphi_0\varphi_c) + \rho_2\varphi_0\varphi_c)\varphi_0\varphi_c}\Big)^{0.5}; \tag{3}$$

$$c_0 = \Big(\frac{P_0}{(\rho_1(1-\varphi_0) + \rho_2\varphi_0)\varphi_0\varphi_c}\Big)^{0.5}. \tag{4}$$

Points 1 and 2 show the results obtained by the homogeneous model (3) and by the cluster model (4), respectively. For low values of the volume content of clusters in the medium φ_0, the velocities of sound predicted by models (3) and (4) are close to each other; correspondingly, the experimental points 1 and 2 lie close for the corresponding wave amplitudes. Curve 3 shows the shock-wave velocity calculated by the isothermal model [3, 4]:

$$U/c_0 = (1 + \Delta P_0/P_0)^{0.5}. \tag{5}$$

The isothermal approximation for the shock-wave velocity (5) offers an adequate description of the experimental data (points 1 and 2). Indeed, if we estimate the time of thermal relaxation of the gas in the bubbles contained in the cluster as $\tau_0 = d^2/(4\pi^2 a)$ [4], we find that the duration of the leading front of the oscillating shock wave is much greater than τ_0 (a is the thermal diffusivity of the gas in the bubbles). Hence, the behavior of the gas in the bubbles in the leading front of the wave is close to the isothermal one. It is impossible to determine which of the models for the velocity of sound, (3) or (4), is in better agreement with the experimental data because the difference between them is within the error of the wave-velocity measurement.

Points 4 and 5 show the experimental values of the shock-wave velocity in the medium with a rather low volume content of the gas, where the liquid compressibility substantially affects the wave velocity. The shock-wave velocity was determined by the time of arrival of the beginning of the leading shock-wave front (points 4) and the peak of the first oscillation (points 5) to two neighboring gauges (G3 and G4). Curves 6 and 7 show the shock-wave velocity calculated with allowance for the liquid compressibility by the adiabatic [22] and isothermal [4] approximations, respectively:

$$\frac{U}{c_1} = \frac{1}{c_1}\Big(\frac{\Delta P_0}{\rho_1(1-\varphi_0\varphi_c)}\Big(1 - \varphi_0\varphi_c\Big(1 + \frac{\Delta P_0}{P_0}\Big)^{-1/\gamma} - (1-\varphi_0\varphi_c)\Big(1 + \frac{\gamma^*\Delta P_0}{\rho_1 c_1^2}\Big)^{-1/\gamma^*}\Big)^{-1}\Big)^{0.5}; \tag{6}$$

$$\frac{U}{c_1} = \frac{1}{c_1}\Big(\frac{P_0 + \Delta P_0}{\rho_1(1-\varphi_0\varphi_c)\varphi_0\varphi_c}\,\frac{1 + \Delta P_0/(\rho_1 c_1^2)}{(1 + \Delta P_0/P_0 - \varphi_0\varphi_c)P_0/(\rho_1 c_1^2\varphi_0\varphi_c)}\Big)^{0.5} \tag{7}$$

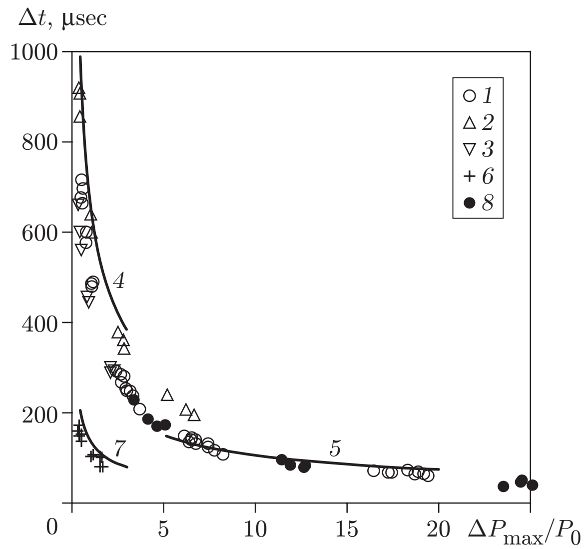

Fig. 4. Half-width of the pressure wave versus its amplitude: points 1–3, 6, and 8 show the experimental data for $\varphi_0 = 7.5\%$ and $\varphi_c = 8.4$–9.8 (1) and 10.3% (2), $\varphi_0 = 18.7\%$ and $\varphi_c = 10.3\%$ (3), $\varphi_0 = 7.5\%$ and $\varphi_c = 0.38\%$ (6), and an isolated cluster for $\varphi_c = 12\%$ (8); curves 4 and 7 refer to the results calculated by Eq. (2) for $\varphi_c = 9.1$ and 0.38%, respectively; curve 5 refers to the calculation by the formula $\Delta t = 0.5\Delta t_{\rm B}$.

(γ and γ^* are the ratios of specific heats for the gas and liquid, respectively). The experimental values of velocity determined by the peak of the first oscillation of the shock wave (points 5) are adequately described by the isothermal approximation (7), whereas points 4 (with the velocity determined by the beginning of the shock-wave front) are located closer to the adiabatic approximation (6). The reason is that the behavior of the gas in the bubbles at the initial part of the leading front of the shock wave is close to the adiabatic one. Deviation of the experimental points 4 from the calculated curve 6 with increasing wave amplitude seems to be related to the decrease in the initial volume content of the gas in the cluster in the course of the experiment, i.e., part of the gas bubbles evolve from the cluster after a shock wave with a fairly high amplitude passes over the liquid, and the value of φ_c decreases.

Points 1–3 and 6 in Fig. 4 show the measured half-width of the first oscillation of the shock wave in a gas–liquid medium with a cluster structure. As for the Boussinesq soliton, the half-width of the first oscillation is the wave length from the level of $0.42\Delta P_{\max}$ to the maximum value of pressure in the first oscillation $\Delta P_{\max}$. The wave half-width was measured only by the gauges opposite the third and fourth clusters counted from the point of the wave entrance into the medium, i.e., when the oscillating wave is almost completely formed. Points 1 and 6 show the data for the cluster positioning in Fig. 1. Points 2 refer to the half-width of the wave from the fourth gauge in a medium without the third cluster. Points 3 refer to readings of the second gauge for a medium with a rather high volume content of clusters (the centers of the clusters are located at a distance of 40 mm from each other, and the first cluster on the top retains its position). Curves 4 and 7 show the soliton half-width calculated by Eq. (2) for medium parameters corresponding to the test conditions. For rather homogeneous media with a cluster structure (with the clusters located uniformly along the test section in the liquid), the half-widths of the first oscillations are identical (points 1 and 3). Deviation of points 2 from experimental data 1 and 3 is apparently caused by the influence of substantial nonuniformity of the cluster structure (the third cluster was removed from the medium). In the case of low wave amplitudes ($\Delta P_{\max}/P_0 \leqslant 1$), as for homogeneous bubbly media, the calculation by Eq. (2) agrees with experimental data for the corresponding parameters of the medium (points 1–3 and curve 4; points 6 and curve 7). Deviation of experimental data from the calculated results is caused, as was noted above, by unsteadiness and violation of the long-wave approximation in experiments.

It was shown [23] that the half-width of solitary waves with the amplitude $\Delta P_{\max}/P_0 \geqslant 5$ in a homogeneous bubbly media is close to $0.5\Delta t_{\rm B}$. The experimental points 1 with the amplitude $\Delta P_{\max}/P_0 \geqslant 5$ for waves in a gas–liquid medium with a cluster structure are also adequately generalized by the numerical dependence 5 ($\Delta t = 0.5\Delta t_{\rm B}$).

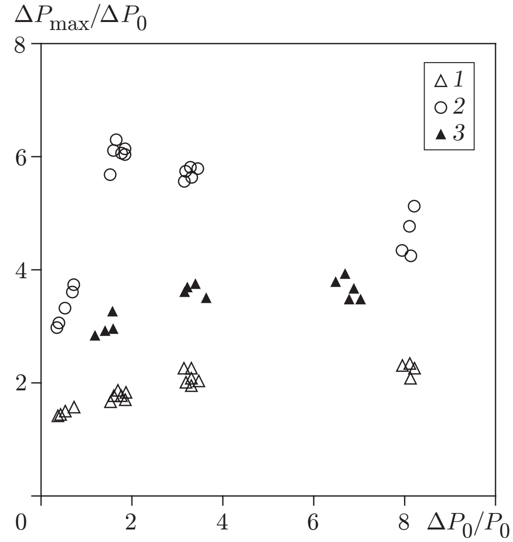

Fig. 5. Amplitude of the first oscillation of the shock wave in a gas–liquid medium versus the amplitude of the shock wave entering the medium: points 1 and 2 refer to the incident and reflected shock waves, respectively ($\varphi_0 = 7.5\%$ and $\varphi_c = 8.4$–9.8%); points 3 show the results for an isolated cluster ($\varphi_c = 12\%$).

For comparison, Fig. 4 also shows the experimental values of the half-width of the solitary wave (points 8) formed in the liquid by an isolated cluster of the same size [16]. The duration of the first oscillation of the oscillating shock wave in a gas–liquid medium with a cluster structure (points 1) equals the duration of the solitary wave formed by an isolated cluster in the liquid (points 8) with identical parameters of the medium and the wave.

The experimental dependences of the amplitudes of the first oscillation of the shock wave (points 1) and the first oscillation of the wave reflected from the rigid wall (points 2) on the amplitude of the shock wave entering the medium are plotted in Fig. 5. The amplitude of the first oscillation of the incident shock wave was measured only by the gauges opposite the third and fourth clusters, i.e., when the oscillating shock wave is almost completely formed. As for homogeneous bubbly media [3, 23], the amplitude of the first oscillation (points 1) is 1.5 times higher than the mean pressure in the shock wave for low wave amplitudes ($\Delta P_{\max}/P_0 \leqslant 1$). With increasing amplitude of the incident shock wave, the values of $\Delta P_{\max}/\Delta P_0$ increase, which is also in agreement with the data of [21] for bubbly media. Note, the experimental values of the amplitude of a solitary pressure wave formed by an isolated cluster (points 3) for the same parameters of the medium and the wave [18] lie substantially higher than points 1. In the case of low wave amplitudes $\Delta P_{\max}/P_0 \leqslant 1$, the amplitude of the wave reflected from the solid bottom is twice the amplitude of the incident wave, which corresponds to a linear reflection law (points 2). As the amplitude of the incident shock wave increases, the reflection law becomes nonlinear.

Let us consider the structure of the pressure wave far from the clusters. In these experiments, the third cluster from the point of the wave entrance into the medium was removed, and the third gauge measured the profile of the pressure wave in the medium at a distance from the neighboring clusters much greater than the cluster size (see Fig. 1). Figure 6a shows the profiles of the pressure waves registered by the gauges located close to clusters (curves 1 and 3) and by the gauge located far from the clusters (curve 2) for a low amplitude of the shock wave entering the medium. It is seen that an oscillating shock wave is formed near the gauges located in a close vicinity of the clusters (curves 1 and 3), whereas the shape of the wave between the clusters corresponds to superposition of waves from the neighboring clusters (curve 2). With increasing amplitude, secondary reflection of waves formed by the neighboring clusters from each other starts to play an important role. As a result, pressure oscillations appear on the wave profiles near the clusters (curves 1 and 3 in Fig. 6b), and the wave profile registered by the gauge between the clusters consists of high-amplitude high-frequency pressure oscillations (curve 2 in Fig. 6b). The period of oscillations corresponds to the time needed for the wave to travel back and forth between the neighboring clusters with the velocity of sound in the liquid.

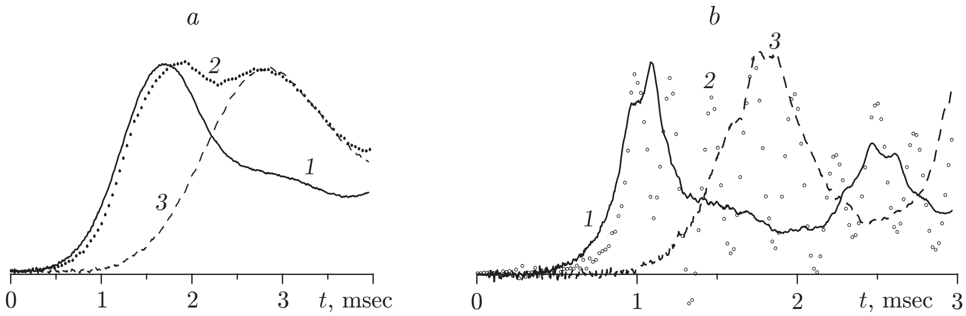

Fig. 6. Structure of the pressure wave in the medium for $\varphi_c = 10.3\%$ and $x = 0.105$ (1), 0.205 (2), and 0.305 m (3): (a) $\Delta P_0 = 0.038$ MPa and $\Delta P_{\max} = 0.071$ (1), 0.044 (2), and 0.046 MPa (3); (b) $\Delta P_0 = 0.2$ MPa and $\Delta P_{\max} = 0.54$ (1), 0.375 (2), and 0.282 MPa (3).

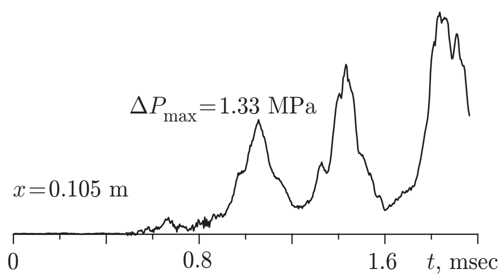

Fig. 7. Amplification of oscillation amplitude in a moderate-amplitude shock wave in a gas–liquid medium with a cluster structure: $\varphi_0 = 11.3\%$, $\varphi_c = 10.8\%$, and $\Delta P_0 = 0.81$ MPa.

It is known that pressure oscillations in an oscillating shock wave in homogeneous bubbly liquids decay behind the leading front of the shock wave [3] because of dissipative losses in the medium. Figure 7 shows the structure of a moderate-amplitude oscillating shock wave in a gas–liquid medium with a cluster structure. A growth of the amplitude of pressure oscillations in the wave behind the leading front is observed instead of decaying oscillations. This can be related to resonant oscillations of clusters in the liquid. Under certain parameters of the medium and the wave, oscillations of three neighboring clusters proceed as follows. The first and last clusters oscillate in phase with each other and in antiphase with the cluster located between them. This increases the amplitude of oscillations of the cluster in the middle and, correspondingly, the amplitude of oscillations in the shock wave.

Thus, experimental data have been obtained on velocity and structure of moderate-amplitude shock waves in a liquid containing spherical bubbly clusters, and a comparison with theoretical models has been performed. For low-amplitude waves, the Boussinesq equation offers an adequate description of the structure of the leading front of the oscillating shock wave. It is also shown that resonant interaction of bubbly clusters in the wave can increase the amplitude of oscillations in the shock wave.

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